

Exercise 23

Prove the statement using the ε, δ definition of a limit.

$$\lim_{x \rightarrow a} x = a$$

Solution

According to Definition 2, proving this limit is logically equivalent to proving that

$$\text{if } |x - a| < \delta \quad \text{then} \quad |x - a| < \varepsilon$$

for all positive ε . Choose $\delta = \varepsilon$. Now, assuming that $|x - a| < \delta$,

$$\begin{aligned} |x - a| &< \delta \\ &= \varepsilon. \end{aligned}$$

Therefore, by the precise definition of a limit,

$$\lim_{x \rightarrow a} x = a.$$